# Horizontal segregation in a vertically vibrated binary granular system

Xiaodong Shi, Guoqing Miao, and Hua Zhang
*Institute of Acoustics and Key Laboratory of Modern Acoustics of the Ministry of Education, Nanjing University, Nanjing 210093, People's Republic of China*
(Received 16 April 2009; published 29 December 2009)

We present numerical simulations and experiments on the horizontal transport and segregation of binary granular mixtures with different sizes and/or different densities in a vertically vibrated container with a sawtooth-shaped base. The larger particles migrate to the positive or negative end of the container, depending on the ratios of the diameters and the densities of two kinds of particles, the vibrating frequencies, and the accelerations. In particular, horizontal segregation occurred even if all the particles have the same size but different densities.

DOI: 10.1103/PhysRevE.80.061306 PACS number(s): 45.70.Mg, 02.70.Ns, 64.75.−g

Granular materials have attracted much attention from scientists in recent years [1–3]. Among them, the vibrated granular materials exhibit very rich phenomena such as fluidization [4,5], convection[6,7], horizontal transport [8–10], heap formation [11], size segregation [12–14], and surface waves [15,16]. The size segregation is an especially interesting and important phenomenon. As a result of advances in numerical simulations of granular materials, scientists are striving for a better understanding details of the movement of the granular materials. Some experimental results in two-dimensional (2D) and three-dimensional (3D) systems have also been qualitatively reproduced by computer simulations[17–19]. Some costly laboratory experiments are especially suitable for replacement by numerical simulations. The results of the computer simulations can also be validated by the experiments.

When a binary granular mixture is vertically vibrated by a flat base, under the proper conditions, the mixture can be segregated. The larger particles rise to the top of the granular layer, called the Brazil-nut (BN) effect, or the smaller particles rise to the top of the granular layer, called the reverse Brazil-nut (RBN) effect [12,13]. When the unitary granular layer is vibrated by a sawtooth-shaped base, the horizontal flow rate varies with the height and the upper and lower parts of the layer can move in opposite directions due to the influence of the sawtooth-shaped base [8,9]. When the granular mixture is vibrated by a sawtooth-shaped base, a stratified flow appears within the layer not only in a vertical direction, but also in a horizontal direction, resulting in horizontal segregation[10]. In this paper we pay attention to the numerical and experimental study of the behavior of a vertically vibrated binary granular system with a sawtooth-shaped base, to explore the effects of several factors, e.g., the sizes and densities of the particles, the driving acceleration and frequency, and the boundary conditions of the system, on the stratified flow and horizontal segregation of the granular system.

We use a two-dimensional system in the simulations and a quasi-two-dimensional system in the experiments. Unlike previous work [10], where all particles have the same density, we use two kinds of particles with different diameters and densities. The sizes of the particles are bimodal and randomly distributed over narrow ranges extending from nominal sizes to values ±5% lower. These particles are confined in a rectangular container with sawtooth-shaped base and reflecting side walls. The container is sufficiently high that particles never reach the upper boundary. The container is oscillated sinusoidally with frequency $f$ and amplitude $A$. The dimensionless acceleration $\Gamma=4\pi^2 f^2 A/g$ (g is gravitational acceleration) is an important quantity in vibrating systems, with $\Gamma \approx 1$ the minimum required to excite the layer. In our simulations, there are 20 identical sawteeth on the base, and in our experiments, there are 26 identical sawteeth on the base, a part of which is shown schematically in Fig. 1. Each tooth can be characterized by three parameters: the height $h$, the width $w$, and the asymmetry parameter $a$ defined as $a=l/w$, where $l$ is the length of projection of the left side of a sawtooth in a horizontal direction. In the experiment, each tooth has $h=3$ mm, $w=6$ mm, and $a=0.90$, respectively. The sidewalls of the container consist of two transparent glass plates with a length of $L=156$ mm, and the width between the plates is $p=9.0$ mm. This meets the condition for the quasi-two-dimensional system (the length is much larger than the width). Glass balls, copper balls, and steel balls are used. In the simulation, the top of each sawtooth is rounded by an arc with radius $s$ which smoothly connects the two sides.

In the simulations, we use the soft-disk model [9,10,18]. Each particle is regarded as a disk with a given radius $r_i$, its

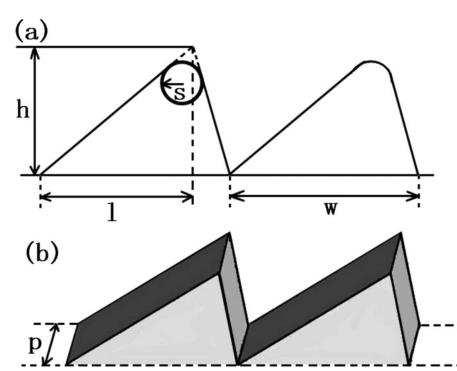


FIG. 1. Schematic diagram of the sawtooth-shaped base. (a) The case for the two-dimensional system. The top of the teeth is rounded by an arc with radius $s$ smoothly connecting to both sides. (b) The case of the quasi-two-dimensional system. The thickness of the sawtooth is $p$.

 

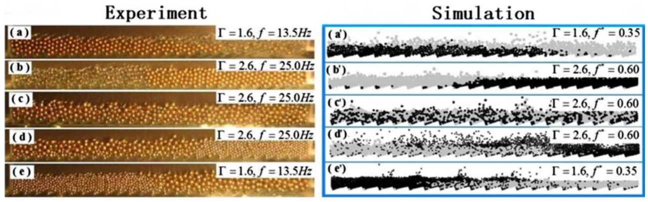


FIG. 2. (Color online) The horizontal segregation of the binary mixture. The left column is the experimental result, and the right column is the numerical result in which the light gray particles are the larger ones and the dark gray particles are the smaller ones. In the experiment, the diameters of the particles are 3.0 (glass) and 2.5 mm (copper) in (a) and (b), 3.0 (copper) and 2.5 mm (steel) in (c), and 3.0 (copper) and 1.5 mm (glass) in (d) and (e), respectively. In the simulation, the diameter ratio of the larger and smaller particles is $d_A/d_B=1.2:1$ in (a′), (b′), and (c′), $d_A/d_B=2:1$ in (d′) and (e′), respectively; the mass ratio is $m_A/m_B=2.5:1$ in (a′) and (b′), $m_A/m_B=1.6:1$ in (c′), and $m_A/m_B=9:1$ in (d′) and (e′), respectively.

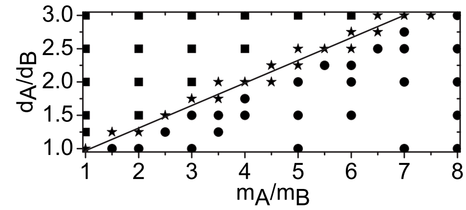


FIG. 3. Phase diagram of the segregation of the binary mixture. The solid squares: the larger particles migrate to the positive side of the container; the solid circles: the larger particles migrate to the negative side; and the stars: two kinds of particles mixed.

moment inertia is $\frac{1}{2}m_i r_i^2$ ($m_i$ and $r_i$ are the mass and the radius of the $i$th particle, respectively). The interaction between two particles has a Lennard-Jones (LJ) form with a cutoff $r_c$ where the repulsive force falls to zero. The repulsive force for particles located at $\mathbf{r}_i$ and $\mathbf{r}_j$ is $\mathbf{f}_{ij}^r=(48\epsilon/r_{ij})[(\sigma_{ij}/r_{ij})^{12}-0.5(\sigma_{ij}/r_{ij})^6]\hat{\mathbf{r}}_{ij}$ for $r_{ij}<r_c=2^{1/6}\sigma_{ij}$, and $\mathbf{f}_{ij}^r=0$ for $r_{ij}>r_c$, where $\mathbf{r}_{ij}=\mathbf{r}_i-\mathbf{r}_j$, $r_{ij}=|\mathbf{r}_{ij}|$, and $\sigma_{ij}=(\sigma_i+\sigma_j)/2$. $\sigma_i$ is the characteristic length representing the size of $i$th particle. In addition, normal and shear dissipation forces produce inelastic collisions and retard sliding during the collision. The normal dissipation force is $\mathbf{f}_{ij}^n=-\gamma_n m_{eff}(\dot{\mathbf{r}}_{ij}\cdot\hat{\mathbf{r}}_{ij})\hat{\mathbf{r}}_{ij}$, and the shear dissipation force is $\mathbf{f}_{ij}^s=-\mathrm{sgn}(v_{ij}^s)\min(\mu|\mathbf{f}_{ij}^r+\mathbf{f}_{ij}^n|,\gamma_s m_{eff}|v_{ij}^s|)\hat{\mathbf{s}}_{ij}$, where $m_{eff}=2m_i m_j/(m_i+m_j)$ is twice the reduced mass of particles $i$ and $j$, $\gamma_n$ is the normal dissipation coefficient, $\gamma_s$ is the tangential dissipation coefficient, and $v_{ij}^s=\hat{\mathbf{r}}_{ij}\cdot\hat{\mathbf{s}}_{ij}+r_{ij}(\sigma_i\omega_i+\sigma_j\omega_j)/(\sigma_i+\sigma_j)$ is the relative tangential velocity of the particle, where $\hat{\mathbf{s}}_{ij}=\hat{\mathbf{z}}\times\hat{\mathbf{r}}_{ij}$ ($\hat{\mathbf{z}}$ is the unit normal to the simulation plane) and $\omega_i$ is an angular velocity. The value of the static friction coefficient is $\mu=0.5$, and the normal and shear dissipation coefficients are $\gamma_n=5.0$ and $\gamma_s=5.0$, respectively. In the simulation, the results are expressed in terms of reduced units in which the length is scaled in unit of the diameter of the particles. The diameter of the particles is about $1.0\times10^{-3}$ m. We set $g=5.0$, then the reduced time unit corresponds to 0.022 s, and the reduced frequency $f^\star=0.022\times f$. Each sawtooth has the height $h_0=2$, the width $w_0=2$, the asymmetry $a=0.90$, and arc radius $s=0.1$, respectively. A very accurate fifth-order predictor-corrector algorithm [20] is used with a time step $\Delta t=0.002$.

Both in the experiments and in the simulations we use $d_A$ and $d_B$, $m_A$ and $m_B$, and $n_A$ and $n_B$ to represent, respectively, the average diameters, masses, and numbers of larger and smaller particles. Then the number of layer is $N_h=(n_A d_A+n_B d_B)/L$. Figure 2 shows a series of snapshots of the experiments and simulations. The horizontal size segregation is evident. The results of experiments and simulations are in close agreement. The larger particles move in a negative direction in (a), (a′) and (e), (e′), and in a positive direction in (b), (b′) and (d), (d′). Here positive direction is defined as follows: moving in the positive direction the first edge of a sawtooth is the steeper one, i.e., from right to the left for the present case. We call these phenomena as the horizontal Brazil-nut effect (larger particles accumulate at the positive side, while smaller ones at the negative side of the container) and the reverse horizontal Brazil-nut effect (large particles accumulate at the negative side, while smaller ones at the positive side of the container). In (c) and (c′), the binary mixtures are hard to segregate; the larger and smaller particles are always mixed.

The results of changing the mass/diameter ratio are shown in Fig. 3. The frequency $f^\star=0.35$ used in the simulations corresponds to 16 Hz, the acceleration $\Gamma=1.6$, and the number of layer $N_h=8$. The total volumes of the larger and smaller particles are the same. The simulation begins with a mixed state of two kinds of particles. The vertical size segregation occurs first; then the particles on the top of the layer migrate to the positive end of the container. But when the larger and smaller particles cannot segregate, the horizontal segregation cannot occur. The results in Fig. 3 are obtained after 10 000 vibration cycles. In general, after 3000 vibration cycles, the horizontal size segregation occurs evidently, and after 9000 vibration cycles, a stable state is reached, the horizontal flow stops. The stratified flow far from the sidewalls has a powerful effect on the separation process. Whether the larger particles migrated to the positive or the negative end of the container depends on the ratio of the diameter and the mass (density) of the larger and smaller particles. But for certain ratio of mass and diameter of the larger and smaller particles, the binary mixtures cannot segregate; they remain a mixture. In particular, we find that even if all the particles have the same sizes but have different densities, segregation still happens.

Here we particularly study the behavior of a mixture consisting of two kinds of particles of the same size but different density. Figure 4 shows a numerical result for the normalized positions of the two centers of mass of all the lighter particles and all the heavier particles after 30 000 vibration cycles. When the number of layers is less than 4, all the particles migrate to the negative side of the container. As the number of layer increases from 1, the position of the center of mass of all the particles gets closer and closer to the center of the container. The higher the $\Gamma$, the closer to the negative end of the container is the center of mass of all particles. When the number of layer increases to and above 4, the net flow rate almost vanishes, but there are still opposing flows within the granular layer; i.e., there is still horizontal segre-

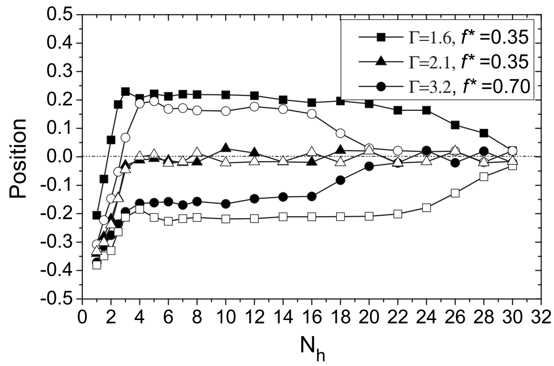


FIG. 4. Positions of the centers of the masses of all the lighter and all the heavier particles as a function of the number of layer. The open markers: the lighter particles; the solid markers: the heavier particles. $d_A/d_B=1:1$; $m_A/m_B=1:5$.

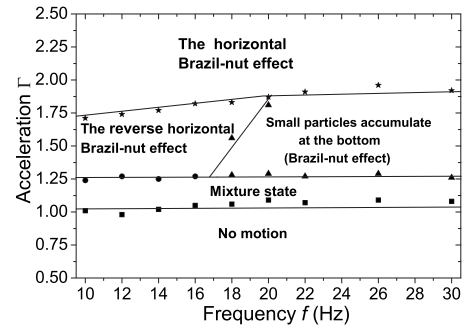


FIG. 5. The state of the system as the driving parameters change. The diameters of particles are 2.5 (copper) and 2.0 mm (glass), respectively. The number of layer $N_h=8$.

gation. Similarly when the acceleration is lower ($\Gamma=1.6$), the center of mass of lighter particles is located at the negative side of the center of mass of heavier particles. When the acceleration increases to $\Gamma=2.1$, the lighter particles are on the top of the heavier particles, both of their centers of mass are close to the center of the container, and there is no horizontal segregation. When the acceleration is higher ($\Gamma=3.2$), the center of mass of lighter particles is located at the positive side of the center of mass of heavier particles.

We have also found some complicated and interesting phenomena within the mixture when driving acceleration and frequency change. Figure 5 is an example of experiment for a mixture consisting of copper balls and glass balls with diameters of 2.5 and 2.0 mm, respectively. The number of layer $N_h=8$. There are five different parameter regions evident in the figure. (1) At lower acceleration ($\Gamma<1.05$), the particles as a solid body vibrate together with the plate. (2) When $1.05<\Gamma<1.26$, the larger particles start to wiggle around their initial positions, many voids are created between larger particles, and the smaller particles move downwards through the voids and fill voids below. (3) As $\Gamma$ increases beyond 1.26, the larger particles reassemble, some large voids are created for a short time, and the smaller particles move freely downwards through these larger voids. At the same time, a stratified flow occurs within the granular layer. Finally, at a lower frequency ($f<16$ Hz), horizontal segregation occurs in which large particles accumulate on the negative side of small ones (i.e., the reverse horizontal Brazil-nut effect). At a higher frequency ($f>20$ Hz), the Brazil-nut effect occurs in which the larger particles move to the top of smaller ones. (4) When $\Gamma>1.65$, the smaller particles tend to move upwards, and the horizontal flow rate increases, a horizontal segregation is evident, and the larger particles accumulate on the positive side of the smaller ones (i.e., the horizontal Brazil-nut effect). In a word, the higher the driving acceleration, the greater the segregation of the mixture. We observe a transition between the horizontal Brazil-nut effect and the reverse horizontal Brazil-nut effect around $\Gamma=1.65$. But the mechanism for this transition is not clear now. The transition between normal Brazil-nut effect and horizontal segregation around $\Gamma=1.5$ in Fig. 5 can be qualitatively accounted for as follows. In order for the mixture to be segregated, the mixture should be fully fluidized and the ratio $\eta$ of average kinetic energy per particle to the potential energy required to overcome a barrier of height $h$ is an important parameter. If we consider $v=2\pi fA$, then $\eta=\langle v^2\rangle/(gh)=\Gamma^2 g/(4\pi^2 f^2 h)$, where $v$ is the velocity of the particle. Using $\Gamma=1.5$, $h=3$ mm (the size of diameter of the particle), and $f=18$ Hz (on the transition line) gives $\eta\approx 0.6$, which is reasonably of order 1.

In summary, using the computer simulations and experiments, we have studied the phenomena of horizontal segregation in detail in a vertically vibrated granular system with a sawtooth-shaped base. The horizontal segregation we observed here is analogous to a vertical one in a vertically vibrating container with a flat bottom, e.g., the horizontal Brazil-nut effect, as we call it, in which the larger particles accumulate on the positive side of the smaller ones, or the reverse horizontal Brazil-nut effect, in which the larger particles accumulate on the negative side of the smaller ones. As the density and diameter ratios changed, the states of the system changed. Especially, we have found that segregation happens even if all the particles are the same size but with different densities. Moreover, through computer simulation, we can predict some new features exhibited by granular systems. Since segregation is essential in the processing of granular materials, our work has potential industrial values.

The authors appreciate Michael Zammett for a critical reading of this paper. This work was supported by the National Natural Science Foundation of China (Grants No. 10474045 and No. 10674067) and the Research Fund for Doctoral Program of Higher Education of China (Grant No. 20040284034).

[1] H. M. Jaeger, S. R. Nagel, and R. P. Behringer, Phys. Today **49**, 32 (1996).
[2] L. P. Kadanoff, Rev. Mod. Phys. **71**, 435 (1999).
[3] P. G. de Gennes, Rev. Mod. Phys. **71**, S374 (1999).
[4] E. Clement and J. Rajchenbach, Europhys. Lett. **16**, 133 (1991).
[5] S. Luding, E. Clement, A. Blumen, J. Rajchenbach, and J. Duran, Phys. Rev. E **49**, 1634 (1994).
[6] P. Evesque and J. Rajchenbach, Phys. Rev. Lett. **62**, 44 (1989).
[7] E. Clément, J. Duran, and J. Rajchenbach, Phys. Rev. Lett. **69**, 1189 (1992).
[8] Z. Farkas, P. Tegzes, A. Vukics, and T. Vicsek, Phys. Rev. E **60**, 7022 (1999).
[9] M. Levanon and D. C. Rapaport, Phys. Rev. E **64**, 011304 (2001).
[10] D. C. Rapaport, Phys. Rev. E **64**, 061304 (2001).
[11] S. Douady, S. Fauve, E. Szmatula, and J. P. Denis, Europhys. Lett. **8**, 621 (1989).
[12] A. D. Rosato, K. J. Strandburg, F. Prinz, and R. H. Swendsen, Phys. Rev. Lett. **58**, 1038 (1987).
[13] D. C. Hong, P. V. Quinn, and S. Luding, Phys. Rev. Lett. **86**, 3423 (2001).
[14] A. P. J. Breu, H.-M. Ensner, C. A. Kruelle, and I. Rehberg, Phys. Rev. Lett. **90**, 014302 (2003).
[15] F. Melo, P. B. Umbanhowar, and H. L. Swinney, Phys. Rev. Lett. **75**, 3838 (1995).
[16] E. Clément, L. Vanel, J. Rajchenbach, and J. Duran, Phys. Rev. E **53**, 2972 (1996).
[17] S. Luding, E. Clément, J. Rajchenbach, and J. Duran, Europhys. Lett. **36**, 247 (1996).
[18] K. M. Aoki and T. Akiyama, Phys. Rev. Lett. **77**, 4166 (1996).
[19] C. Bizon, M. D. Shattuck, J. B. Swift, W. D. McCormick, and H. L. Swinney, Phys. Rev. Lett. **80**, 57 (1998).
[20] M. P. Allen and D. J. Tidesley, *Computer Simulations of Liquids* (Oxford University Press, Oxford, 1987).